





Problem

Problem: separate data into similar groups -> Clustering Solution: Visualize data in two dimensions as landscape -> Projection Goal: detect and visualize meaningful cluster structures



Common Concepts of Swarms

Swarm intelligence
 [Beni 1989], [Cao et al., 1997],
 [Grosan et al., 2006]

"the collective behavior of many simple entities called agents"

Self-Organization
 Swarms: [Bonabeau/Dorigo et al., 1999]
 SOM: [Ultsch 1992]

"spontaneous pattern formation by a system itself, without responsibility of any determinate inside agent"

2. Bionics

[Deneubourg 1991, Reynolds, 1987]

"the application of biological methods and systems found in nature"

My Ideas

- Combine these three concepts
- Make use of Game theory
- Make use of Emergence

the arising of novel and coherent properties during the process of selforganization

Prior: DataBots [Ultsch 1999]



jeder DataBot repräsentiert **genau einen** Datensatz

zt auf einer Gitterposition fänglich zufällig)

urteilt seine Umgebung

sitzt Bewegungsstrategien für jede ter Richtung

Bewegt sich in Richtung Verbesserung

Introduction into Game theory

- decision-making situations are modeled in which several parties influence each other
- Derive rational decision-making behavior in social conflict situations by using mathematical mpodels
- Used in economics (Operations Research), political science, and psychology, biology and poker
- 8 Nobel Memorial Prices
- Important representatives: John von Neumann and Oskar Morgenstern ("Theory of Games and Economic Behavior"), John Forbes Nash Jr, Thomas Schelling, Daniel Kahneman
 - RAND Corporation (offer research and analysis to the United States Armed Forces -> space race, nuclear deterrence in Cold war)
- Swarm Intelligence <-> Game theory [Thrun, 2016]

Game theory

- A game is defined by i=1...n players, where each player makes a choice P
- The choice of each player determines the outcome for each player
 in general the outcome will be different for different players

=> In a game, the payoff λ_i of each player i depends not only on his own choice, but also on the choice of all other players

=> models situations in which multiple players interact or affect each other's outcomes

Game theory

- the choices P_i, i = 1, ..., n are defined as a set of mixed strategies b_i for each player i
- A strategy $b'_i \in P_i$ is the equilibrium, if no deviation in strategy by any single player is more profitable for that player *i*
- For non-cooperative games [J. Nash, 1951] proved the existence of at least one equilibrium point:

 $\forall i, \qquad b_i \in P_i: \lambda_i(b'_i) \ge \lambda_i(b_i)$

Examples

⇒ a solution concept always exists for egoistic agents in a non cooperative game

What does this mean?

-> watering garden during drought

What would be an quilibirum for a game?

-> Prisoner's dilemma

watering garden during drought

- Should the garden be watered?
 - self-interest (payoff: living garden) <-> restraint (conserving water for community)
- No matter what the other persons do, a person is always better off watering his garden:
 - unnecessary for one person to exercise restraint if the most other persons are restraining as well
 - Even if the rest of the community doesn't exercise restraint, it is futile for just one person to do so since one person does not have that big of an impact on the whole water supply

• Cooperative game: e.g. fine for watering the garden

Prisoner's dilemma - requirements

- Two persons A and B imprisoned for a crime.
- Each prisoner is in solitary confinement with no means of communicating with the other.
- The prosecutors lack sufficient evidence to convict the pair on the principal charge
- Decision of each person does not affect future reward/punishment

Prisoner's dilemma - choices

- 1. If A and B each betray the other, each of them serves 2 years in prison
- 2. If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa)
- 3. If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)

Rational solution: (3)

Nash equilibirium: (1)

Details see: ../Subversion\WGR\Hausarbeiten\16Spieltheorie

Concepts for DataBionicSwarm (DBS)

- Swarm intelligence
- Self-Organization and Emergence
- Bionics
 - 1. Communication: Scent
 - 2. Living in and moving on a flat surface
 - 3. "Ants" wear their colony's dead -> *will wear data points*
 - 4. Preference: Smelling the surroundings of ones place
 - 5. "Ant" moves to a free positions, if it prefers the scent of the new position
- Application of Game Theory
 - Non-cooperative game -> Nash equilibrium

How to describe the preference for having neighbors of their own type?

Requirements for *preference*

- Topoligical neighborhood and distance on grid
- Correct position determination
- Requirements for projection method
 - Focussing scheme with proper reduction of neighborhood -> Nash equilibrium

physics:

- Given problem -> Search first for symmetry
- Solution results in parameter reduction
 - □ Rmin, Rmax, eppochs, ...
 - □ Neighborfunction
 - □ Grid form and grid distance

Databionic Swarm Principle

- A projection based clustering method based on emergent, self-organizing, egoistic, artificial life forms
- The collective is called DataBionic Swarm (DBS)
 - Every entity is defined as a DataBot
- Every DataBot is represented by ex one data point
- DataBots communicate through stigmergy:
 - 1. live on a two-dimensional toroid gi an environment
 - 2. are able to smell a position and er scent



DBS has three interchangeable modules

- I. Polar swarm (Pswarm)
 - □ symmetry considerations
 - Polar Coordinates on a Hexadiagonal toroidal grid
 - Adept definition of distance in two dimensions
 - □ Application of game theory: Scent -> payoff
 - □ Annealing scheme based on game theory
 - Searches for Nash equilibirum in each eppoch
 - Emergence: No Objective function or parameters (Except R^n distance)
- II. Visualization: topographic map with hypsometric tints
 - □ Simplified ESOM-Algorithmus for Interpolation, parameter- free
 - □ Automated U*matrix computation
 - □ 3D landscape generation
- III. Semi-automated Clustering

Scent of DataBots == payoff in game theory

Hermann 2009 (SOP):

Let D(I,j) be the distance of $x_i, x_j \in I$, let d(I,j) be the distance in the Output space O, let F_R : $R \rightarrow [0,1]$ be an arbitrary but continuous and monotone decreasing function, then the scent $\lambda(b_j, R)$: $\mathbb{R}_0^+ \times O \rightarrow \mathbb{R}_0^+$ is defined with

Prefernce/Scent:
$$\lambda(b_j, R) = \frac{\sum_{l \in I} F_R(d(j,l)) * D(j,l)}{\sum_{l \in I} F_R(d(j,l))}$$

DataBionicSwarm

Payoff:
$$\lambda(b_j, R, S_0) = \begin{cases} S_0 - \frac{\sum_{l \in I} H_R(r(j,l)) * D(j,l)}{\sum_{l \in I} H_R(r(j,l))}, & iff \sum_{l \in W} H_R(r(j,l)) > 0 \\ S_0, & else \end{cases}$$

Scent of DataBots == payoff in game theory

Hermann 2009 (SOP):

Let D(I,j) be the distance of $x_l, x_j \in I$, let d(I,j) be the distance in the Output space O, let F_R : $R \rightarrow [0,1]$ be an arbitrary but continuous and monotone decreasing function, then the scent $\lambda(b_j, R)$: $\mathbb{R}_0^+ \times O \rightarrow \mathbb{R}_0^+$ is defined with

Maximizes

structure preservation

Prefernce/Scent:
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DataBionicSwarm

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DataBionicSwarm

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Algorithm

function Positions O=pswarm(matrix D(l,j))

for all $z_i \epsilon$ I: assign an initial random polar position $i_{\phi}(r) \epsilon$ O on the grid

to generate DataBots $b_i \in B$

for R={Rmax=Lines/2,...,Rmin} do

calculate chance c(R)

Repeat for each iteration

c = sample(c(R), B) $m_{\tilde{i}}(c) = uniform(1, Rmax), \tilde{i}=1,...,\alpha$ $l(c) = argmax_{j}(\lambda(b_{j}, R)) \text{ with } j = \{i, m_{\tilde{i}}(c)\} \in O$ l(!c) = i $S = \sum_{l} \lambda_{l}(b_{l}, R)$ $until \frac{\partial S(e, \lambda(R))}{\partial e} = 0$

return O in cartesian coordinates

end function pswarm Thrun - Databionics University of Marburg

Algorithm

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Grid size automatically chosen, contrary to ESOM

return O in cartesian coordinates

end function pswarm Thrun - Databionics University of Marburg

Details: Define Gridsize

Let the size of the grid defined by Lines L and Columns C, N number of DataBots, α be the number of possible jumping positions, $\beta \in (0.5,1]$, $p_{01}(D)$ robust minum, $p_{99}(D)$ robust maximum

I.: $\frac{\sqrt{C^2 + L^2}}{1} \ge \frac{p_{01}(D)}{p_{99}(D)} =: A$ shortest/longest distances assignable to grid units **II**.: $L * C \ge \alpha * N$ enough free positions for a DataBot to jump **III**.: $\frac{L}{C} = \frac{\beta}{1}$ rectangular grid

Resulting in a bi-quadratic equation of $C^4 - A^2 * C^2 + \alpha^2 * N^2 = 0$

$$z_{1/2} = A^2 \pm \frac{1}{2} \sqrt{A^4 - \frac{\alpha^2}{4} N^2}$$

=>
$$C = \begin{cases} \frac{1}{\sqrt{2}} \sqrt{A^2 + \sqrt{A^4 - \frac{\alpha^2}{4}N^2}}, & \text{If } A^4 > \frac{\alpha^2}{4}N^2 \\ approximation, else \end{cases}$$

Algorithm

function Positions O=pswarm(matrix D(l,j))

University of Marburg

for all $z_i \epsilon$ I: assign an initial random polar position $i_{\phi}(r) \epsilon$ O on the grid

to generate DataBots $b_i \in B$ for R={Rmax=Lines/2,...,Rmin} do calculate chance c(R) Repeat for each iteration Parameter c = sample(c(R), B)free $m_{\tilde{i}}(c) = uniform(1, Rmax), \tilde{i}=1,...,\alpha$ annealing $l(c) = argmax_i(\lambda(b_i, R))$ with $j = \{i, m_i(c)\} \in O$ process l(!c) = i $S = \sum_{l} \lambda_{l}(b_{l}, R)$ until $\frac{\partial S(e,\lambda(R))}{\partial e} = 0$ return O in cartesian coordinates end function pswarm Thrun - Databionics

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Details: annealing process

Precise neighborhood function (polar (r,phi) coordinates)
Symmetry considerations (physics): d(j,l) -> radius between I and j and

$$F_{R} = H_{R} = \begin{cases} 1 - \frac{r(j,l)^{2}}{\pi R^{2}}, & iff \ \frac{r(j,l)^{2}}{\pi R^{2}} < 1\\ 0, & else \end{cases}$$

- Rmax=Lines/2 else self interaction
- Rmin >>1 else "ant mill" [Schneirla 1971]
 - □ Fixed
 - Determined by grid size,
 - Number of DataBots



Algorithm

function Positions O=pswarm(matrix D(l,j))

for all $z_i \epsilon$ I: assign an initial random polar position $i_{\phi}(r) \epsilon$ O on the grid

to generate DataBots $b_i \in B$ for R={Rmax=Lines/2,...,Rmin} TradeOff do calculate chance c(R)Batch vs Online Repeat for each iteration c = sample(c(R), B) $m_{\tilde{i}}(c) = uniform(1, Rmax), \tilde{i}=1,...,\alpha$ $l(c) = argmax_i(\lambda(b_i, R))$ with $j = \{i, m_i(c)\} \in O$ l(!c) = iSearching for $S = \sum_{l} \lambda_{l}(b_{l}, R)$ Nash until $\frac{\partial S(e,\lambda(R))}{\partial e} = 0$ equilibirum return O in cartesian coordinates end function pswarm

Algorithm

function Positions O=pswarm(matrix D(l,j))

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for all $z_i \epsilon$ I: assign an initial random polar position $i_{\phi}(r) \epsilon$ O on the grid

to generate DataBots $b_i \in B$ for R={Rmax=Lines/2,...,Rmin} do Choice is set calculate chance c(R)of mixed strategies Repeat for each iteration c = sample(c(R), B) $m_{\tilde{i}}(c) = uniform(1, Rmax), \tilde{i}=1,...,\alpha$ $l(c) = argmax_i(\lambda(b_i, R))$ with $j = \{i, m_{\tilde{i}}(c)\} \in O$ l(!c) = i $S = \sum_{l} \lambda_{l}(b_{l}, R)$ Egoistic, nonuntil $\frac{\partial S(e,\lambda(R))}{\partial e} = 0$ cooperative game return O in cartesian coordinates end function pswarm Thrun - Databionics



Example:

- Choice of projection method
- Choice of parameters for the projection

Structure preservation: projection $R^N \rightarrow R^m$ with N > M cannot preserve all distances!



DataBionic Swarm II

Contains:

- I. Pswarm
- II. Visualization: topographic map with hypsometric tints
 - 1. Transformation from hexagonal grid to BestMatches
 - 2. Simplified ESOM-Algorithmus for Interpolation
 - 3. Automated estimation for P-Matrix Radius [Thrun et al. 2016]
 - 4. U*matrix computation [Ultsch et al. 2016]
 - 5. 3D landscape generation [Thrun et al. 2016]
 - 1. Normalization of U*-Matrix + automatic rectangular Island generation
- III. Semi-automated Clustering
 - □ Uses abstract U-distances [Lötsch/Ultsch, 2014]
 - □ Hierarchical Clustering [Ultsch et al. 2016]

Visualization technique: topographic map with hypsometric tints

U*-Matrix Vizualization of Pswarm:

considers distance and density based structures of data
 Reason:

take the structures seen on the U-matrix into account, s.

Lötsch, J., Ultsch, A.: Exploiting the structures of the U-matrix, 2014

- -> Clustering:
- Formalization of the structures through a U-matrix such that
 - □ distance calculations between best-matching units
 - \Box the height structures of a U-matrix (U-cell distance).
- This enables
 - the assessment of structure preservation and the implementation of clustering algorithms.

Simplified ESOM-Algorithmus for Interpolation

Two Problems:

1.)

ESOM algorithm is about a Best Matching Units (BMUs) search But Pswarm results in given BMUs

2.)

In the normal ESOM algorithm following parameters have to be set

- Neighborfunction
- maxEppochs
- AnfangsLernrate
- EndLernrate
- CoolingMethod
- AnfangsRadius
- EndRadius
- Lines and Columns

Detail1: ProjectedPoints to BestMatches

Let the Grid begin on the x-axis at 1 and end with a maximal number defined as Columns, similar for Lines, then

$$\frac{Lines - 1}{Columns - 1} \approx \frac{|\max(y) - \min(y)|}{|\max(x) - \min(x)|} = \frac{dy}{dy} = \Delta$$

 describes, that the grid size should equal the size of the coordinate system as accurate as possible

The grid size should be around 4096:

Lines * *Columns* \ge 4096

• The resulting equation to be solved is $Lines^2 + Lines(1 + \Delta) - 4096\Delta \ge 0$

Resulting in

$$Lines \geq -\frac{1+\Delta}{2} + \sqrt{\left(\frac{1+\Delta}{2}\right)^2 + 4096\Delta}$$

sESOM: Special Case of ESOM

Idea: Let the bestmatches be fix [Mörchen/Utlsch 2005] My idea: with good neighborhood function the algorithmus can be simplified

The neighborhood function ${\boldsymbol{\mathsf{h}}}$ is defined by

$$\begin{split} H_R &: R \to [0,1] \\ h = \begin{cases} 1 - \frac{d(j,l)^2}{\pi R^2}, & iff \ \frac{d(j,l)^2}{\pi R^2} < 1 \\ 0, & else \end{cases} \end{split}$$

- maxEppochs=10
- Learning rate not required (==1)
- anealingScheme='linear' (and only for Radius)
 - CurrentRadius = max(AnfangsRadius-CurrentEpoch,1)
 - □ AnfangsRadius, EndRadius well and autimatically defined

Simplified ESOM (sESOM)

Initialisierung

Setze An BMU Positionen der Karte die Gewichtsvektoren = Datensätze

Für jede Eppoche

Führe ESOM-Algorithmus mit oberen Parametern ohne Suche nach bestmatches aus

Ersetze an BMU Positionen der Karte die Gewichtsvektoren= Datensätze

U-Matrix: Standard über Moore-Nachbarschaft

Details: P-Matrix Radius Abschätzung

- Schätze minimalen, korrekten Radius und maximal möglichen Radius über ABCanalyse[Ultsch/Lötsch 2014] der Distanzen ab.
- u.U. Korrigiere Interaktiv an iPmatrix() nach
- P=InterClusterdistanz/InnerClusterdistanz
- InterClusterdistanz=min(Distanz(GruppeA))
- InnerClusterdistanz=max(Distanz(GruppeC))

U*matrix

- 3D landscape defined by U*matrix [Ultsch et al., 2016]
 - Combines Umatrix and Pmatrix
- Umatrix: folding of high dimensional space [Ultsch/Siemon, 1990]

(-) In literature cited as grey-scaled 2D vizualization (e.g. [Kadim Tasdemir/Merényi, 2012])

-> Precise colored definition required

- Pmatrix: high dimensional density estimation technique [Ultsch, 2003]
 - (-) Estimation for hypersphere of radius is trying

-> use ABCanalsys [Ultsch/Lötsch, 2015]

=> U*matrix represents distance and density based structures!

U*matrix -> 3D landscape

use colors proposed in [Ultsch, 2003]

=> hypsometric tints: surface colors which depict ranges of elevation

- Calculate contour lines
 - □ Normalization of U*heights, define height intervalls, ...
- Combine specific color scale with contour lines
- Create island of toroid map (rectangular)

=> topographic map with hypsometric tints

(DBS) DataBionic Swarm III

- I. Pswarm
- II. Visualization: topographic map with hypsometric tints
- III. Semi-automated Clustering
 - Uses abstract U-distances [Lötsch/Ultsch, 2014]
 - □ My idea: Calculate Shortest Paths between these distances -> new distance matrix \widehat{D}
- Apply Hierarchical Clustering [Ultsch et al. 2016] to \widehat{D}
- My Idea: Only single linkage (SL) or ward relevant:
 - □ See Compact versus connected structures.
 - -> Cluster analysis requires the choice of compact or connected as a parameter
- Second parameter: number of clusters
 - □ Estimate from dendrogram oder visualization

Abstract U-distances

- Delaunay Graph: Graph of Voronoi cells
 - A region corresponds to each BestMatch consisting of all points closer to that BestMatch than to any other
- Delaunay Path: Number of edges between two BestMatches
- Djikstra Shortest Paths of Delaunay graph weighted with highdimensional Distances

Reason:

- U-matrix is approximation of Abstract Umatrix [Lötsch/Ultsch 2014] which is based on Voronoi cells
- Height of Voronoi-Borders=Euclidean
 High-dimensional distance of data



Übernommen von Prof. Ultsch 40

Hierarchical Clustering

- Clustering distance= shortest paths
- Aggloremative:
 - beginning: every point is one cluster
 - Calculate "similarity" between clusters
 - Unite the two most "similar" clusters to new cluster
 - Do this until only one cluster exists
- Similarity of clusters: depends on algorithm
- Evaluation of number of clusters:
 - Dendrogram -> "ultrametric"





Verification of DBS

- No general error for projections exists (Thesis)
- Statistical: Only through accuracy of given Classification possible
 - Natural datasets: may have different clusters depending on goal
 - =>Artifical data sets: Fundamental Clustering Problem Suite (FCPS)
- Visual: 3D Landscape of known datasets
- Application based: Searching for new Knowledge

Visual Benchmarking on FCPS

Using 3d landscape: Topogrpahic map with hypsometric tints

Data/Projectio n	ESOM	NerV	SOP	DBS
Atom (k=2)	Trial and par- ameter depended	OK	3 Clusters	OK
Chainlink (k=2)	OK	OK	(OK)	OK
EngyTime (k=2)	Clusters invisible	PCA init	Intermixed Clusters	OK
Lsun (k=3)	(OK)	PCA init	Ok?	OK
Golfball (k=1)	OK	Clusters visible	OK	OK
WingNut	Too many substructures	PCA init	Ok?	OK
Other FCPS data sets	OK	OK	OK	OK

Example: EngyTime ESOM



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Example: EngyTime SOP







Accuracy of DBS Clustering

Error-Range of Common Clustering Algorithms



Application I: High-dimensional Data

Leucemia

554 Patients with 7447 active Genes

Four diagnosis

6 Klassen reproduzieren
 Die Vorklassifizierung mit einer Accuracy von 99.6%

Two Outliers:

-> problem in diagnosis

-> see our future publication [Brendel, et al.,2017]



Application 2: Genetic Data of Bees

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98 99

Main Clusters depend on Location of Bees



Outliers are in Queensland



Knowledge Discovery with DBS I



Daily courses of 14 Variables over 2 years are analyzed

Five clusters are shown in the visualization of DBS



Thrun - Databionics University of Marburg

[CIs No 1 | [CIs No 2 | [CIs No 3 | [CIs No 4 | [CIs No 5 |

Clusters explained I

- Apply Cart-Tree to clusters: 99% of data correctly classified
- Knowledge aquisition from a Carrt Tree results in 5 classes

No. of incorrect classifications/No. of observations



Clusters explained II

Clusters can be explained by electric conductivity in the stream

=> Define water quality => allow Prediciton

I



Knowledge Discovery with DBS II (a)

- 535 genes of Pain
- Additional information from GeneOntology data base
 => Define tf-idf distances



Knowledge Discovery with DBS II (b)

- Use in Geneset of each cluster of Overrepresentation anaylsis (ORA)
- Filter pValues with ABCanalysis
- Interpret the directed acyclic graph (DAG)
 - ⇒ Reproduces knowledge of [Lötsch et al., 2013
- Outliers from "noise"
- To new functional areas found:
 - Creatine metabolic process
 - hematopoietic stem cell differentiation



Summary

- DataBionicSwarm (DBS) is an interactive Visualization and Clustering approach consting of 3 modules
 - Parameter free
 - □ Independ of number of variables
 - □ Self-organizing and emergent
- DBS uses projection and high-dimensional data for clustering
- Every module is interchangable with another state of the art application of each respective field, e.g.
 - Projection: NeRV instead of Pswarm
 - Clustering: Spectral clustering instead of DBSclustering

Future Work

- find a strong Nash equilibrium
- apply, connections to solid-state physics
 - E.g Use Bravais lattice or apply a Fourier transformation to the reciprocal lattice (do calculations in the reciprocal space)
- Abtract U*-distances instead of abstract U-distances
 - □ How to integrate Pmatrix information?
- Add new data incremently to given projection
 Efficiency: Project more than 4000 points in less than 24h
- Better Knowledge Extraction from Cluster structures

Thank you for listening